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KINETIC MODEL OF SPALLING FRACTURE

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The theory of longevity based on thermofluctuational representations [1] has received considerable development at this time. It is shown that the thermofluctuational mechanism of fracture is conserved in a longevity time band from several years to fractions of a microsecond.

The S. N. Zhurkov formula

$$t_p = t_0 \exp\left\{\frac{U_0 - \alpha\sigma}{k\vartheta}\right\} \qquad (1)$$

is a classical dependence of the longevity t_p on the load, where $t_0 \approx 10^{-13}$ sec, k is the Boltzmann constant, ϑ is the absolute temperature, σ is the tensile stress, U_0 is the activation energy of the fracture process, and α is a structural parameter of the material.

However, the possibilities of practical application of (1) are limited because the parameters U_0 and α turn out to be dependent on the loading conditions (the kind of stress state, the loading mode, etc.). These limitations can be reduced to a significant degree if, as is customary in mechanics [2, 3], differential equations for the development of vulnerability, particularly those that would yield a dependence close to the S. N. Zhurkov formula for the longevity as solutions for the case of one-dimensional tension on a rod, were used to perform the computations.

Equations of a similar kind were proposed in [4, 5] for the one-dimensional and volume states of stress. Comparing the computation results with experimental data shows that the equations yield the regularities of the development of material vulnerability sufficiently completely in different loading modes.

In the interests of simplification, the one-dimensional case is examined in this paper, and the equation

$$d\omega/d\tau = (1 - \omega)Sh\{\varphi(X/(1 - \omega))\} \qquad (2)$$

is used to perform the computations, where ω is the material vulnerability ($0 \leqslant \omega \leqslant 1$); τ is the dimensionless time introduced in place of the time t by using the formula $\tau = \nu t$; $\nu = \nu_0 e^{-Y}$; $Y = U_0/k\vartheta$; ν_0 is a material constant; $X = \alpha\sigma/k\vartheta$ is a dimensionless load parameter; and U_0, α, k, ϑ, σ retain the same meanings as in (1).

The factor $(1 - \omega)^{-1}$ in the argument of the function φ takes account of the rise in the mean stresses in the damaged section.

In conformity with the general representations [1], we assume the activation energy U of the fracture processes to vary nonlinearly as a function of the applied stress σ. The general view of the dependence $U(\sigma)$ and its approximation by piecewise-linear functions are shown in Fig. 1.

In performing the computations below, we used the dependence $\varphi(X/(1 - \omega))$ that describes the reduction of the activation energy of the fracture processes $U = U_0 - \varphi(\sigma)$ from its initial value U_0 in the form of a piecewise-linear function "without strengthening," which recalls the strain diagram of an ideally plastic material in its form:

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Fig. 1

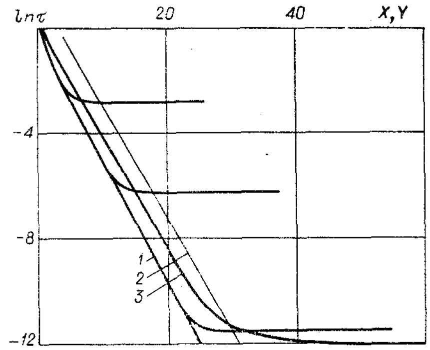

Fig. 2

$$\varphi(X/(1-\omega)) = \frac{X}{1-\omega} \quad \text{for} \quad X/(1-\omega) < Y,$$
$$\varphi(X/(1-\omega)) = Y \quad \text{for} \quad X/(1-\omega) \geqslant Y. \tag{3}$$

The material longevity τ computed by means of (2) and (3) and determined from the condition $\omega(\tau) = 1$ is presented in Fig. 2 (curve 1) as a function of the values of the parameters X and Y for the case of a load constant in time (X = const). For values of the parameter X in the range $0 < X < Y$, a significant quasilinear value corresponding to the Zhurkov formula is observed on the computed curves. For high values of the parameter X the insertion of the constraint Y on the function $\varphi(X/(1-\omega))$ in (2) results in a corresponding lower bound on the longevity level whose limit values for the loading mode under consideration are shown by the line 2 in Fig. 2 as a function of values of the parameter Y (X = const).

The insertion of a "strengthening" section in the function $\varphi(\sigma)$ after the breakpoint in the $\varphi(\sigma)$ diagram results in the appearance of an appropriate second quasilinear section on the material longevity curve (in the X = const regime) instead of the "plateau" zone that appears on the curve $\tau(X)$ (Fig. 2) in the presence of the constraint $\varphi(X/(1-\omega)) \leqslant Y$.

It is known [1, 6] that the process of material fracture occurs in several stages in time: the stage of generation and development of defects and microcracks, during whose initial period the logarithm of the vulnerability growth rate is proportional to the applied stress, and the concluding stage called athermal or sonic, when macrocrack propagation proceeds at approximately constant velocity, comprising about half the velocity of the shear wave in the material.

The selection of the dependence $\varphi(\sigma)$ in the form of a bounded piecewise-linear function permits compliance with the vulnerability growth conditions in both stages.

In fact, we obtain

$$\ln \frac{d\omega}{dt} \approx \ln \nu + \ln Sh(X) \approx \ln \nu + \frac{\alpha}{k\vartheta}\sigma$$

from (2) and (3) for the initial period of vulnerability development ($\omega \approx 0$) under loads $X < Y$.

The vulnerability ω in the material is hence accumulated in proportion to the time of the process. The deviation of the dependence $\omega(\tau)$ from the linear becomes noticeable, as the computations performed show, for a vulnerability comprising $\omega \approx$ 0.03-0.05 as a function of the load parameter X.

The time when the load reaches its limit value Y in the damaged section can be identified with the beginning of the athermal fracture stage since the rate of vulnerability accumulation will here be determined, in practice, by the magnitude of the structural parameter ν_0:

$$\ln(d\omega/dt) = \ln \nu_0 + \ln(1-\omega) + \ln\{e^{-Y} Sh(Y)\} \approx \ln \nu_0.$$

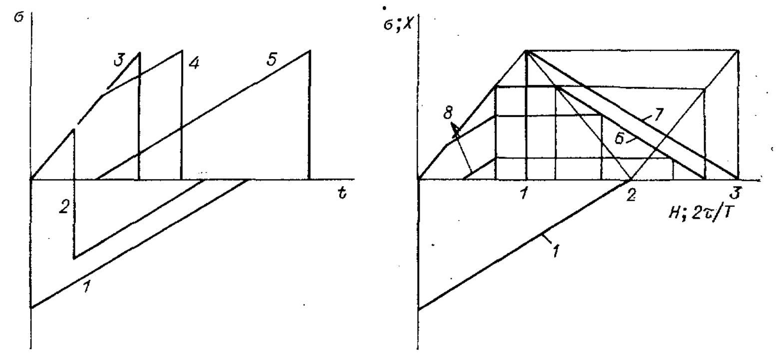

Fig. 3

Specific values of ν_0 and U_0 are determined experimentally for each material, which permits relating the characteristics of the macroprocess being observed to the deeper physical parameters of the fracture phenomenon.

Naturally it is impossible to apply (2), using a homogeneous stress distribution in the section and assuming a homogeneous nature for the fracture of the volume, directly to the determination of the time of microcrack passage across a section of specific dimensions. To perform such a computation, it is necessary to introduce, in addition, the stress distribution in the plane of the crack as is done in the mechanics of cracks, and all the more so since known experimental data on the dependence of material longevity on the stress intensity factor [6, 7] do not contradict the thermofluctuational approach.

However, an equation of the type (2) can be applied to describe another high-speed process, spalling fracture, since in this case the process of vulnerability development is almost homogeneous as is noted in [8] (microcracks develop from a large number of foci separated by 0.1-1.0 mm spacings).

To this end, let us consider a straight rod of constant cross section to whose free end a compression pulse in the form of a right triangle with diminishing amplitude is delivered.

After the beginning of reflection and the time dependence of the tensile stresses in rod sections with the relative coordinate $H = 2\nu S/CT$ (C is the velocity of impulse propagation, T is its duration, and S is the distance along the axis from the free end of the rod), the impulse profile can be found by using the constructions presented in Fig. 3 (1-5 are the stress diagrams in the rod for $\tau = 0$, $\tau < T/2$, $\tau = T/2$, $T/2 < \tau < T$, $\tau > T$, respectively, 6, 7 are the time dependence of the tensile stress in the sections $H < 1$ and $H = 1$, and 8 is the shape of the spalling impulse). For a compression pulse in the form of an arbitrary triangle, the tensile stress diagrams in the sections retain the shape presented in Fig. 3, with the sole difference that the domain of constant stresses become nonsymmetric relative to the axis H = 1.

Curve 1 in Fig. 4 sets up the dependence between the amplitude X and the duration T of the tensile linearly decreasing load for H = 1, the remaining curves refer to the case under consideration of reflection of a compression pulse from the free endface of a rod, and characterize the relation between the amplitude X of the fracturing pulse and the time τ_c to fracture of a rod section with the coordinates H = 1.0 – 2; 0.95 – 3; 0.9 – 4; 0.8 – 5; ... ; 0.1 – 12. The pulse magnitude $I = XT/2$, corresponding to H = 1, is hence minimal for a given amplitude X since the rise in vulnerability from zero to one is here achieved in a time equal to T. Increasing the amplitude of the pulse X while conserving its duration T results in diminution of the coordinates of the spall section (dimensionless H and dimensional S).

The amplitudes of the compression pulse for which spall occurs in sections with a given value of H are determined numerically on the basis of the condition of minimum total time to fracture the section $\tau_c = \tau_h + \tau_p$ ($\tau_h = TH/2$ is the time of origin of the tensile stresses in the section H and $\tau_p = \tau_p(X, H)$ is the material longevity for a change in stress according to the law $X = X(H)$ corresponding to a given H, see Fig. 3).

Investigation of the influence of pulse duration and amplitude on the state of material vulnerability in the spall zone is of interest.

The rod vulnerability at the time of fracture is noted by 1-3 in Fig. 5 for pulses with the amplitudes X = 8, 16, 28 and appropriate minimal durations, 4-6 are the same with the pulse duration doubled, 7 is for an approximately double pulse amplitude relative to the initial value of X = 8 (here H = 0.7).

It is impossible to make any final conclusions about vulnerability from the results presented in Fig. 5, or as it is natural to assume, about the related characteristics of material breakup since the possibility of the appearance of repeated spalls after unloading the material in the spall section was not provided in the calculation program. However, it is already seen from the results obtained that the pieces being spalled will be damaged least if the spalling is performed under the condition H = 1, i.e., for a minimal pulse amplitude (for fixed T). For spalling with H < 1 a zone of large damage will be in front of the spall plane, and whose presence can generally result in an incorrect interpretation of the test results.

The nature of the vulnerability depends on the relationship between the pulse amplitude X and the limit parameter Y as the pulse duration increases. If spalling is performed for a relationship $X/Y < 1$, then the fracture zone is strongly localized, while for $X/Y \approx 1$ the material is damaged intensely in a significant length. These results, obtained on the basis

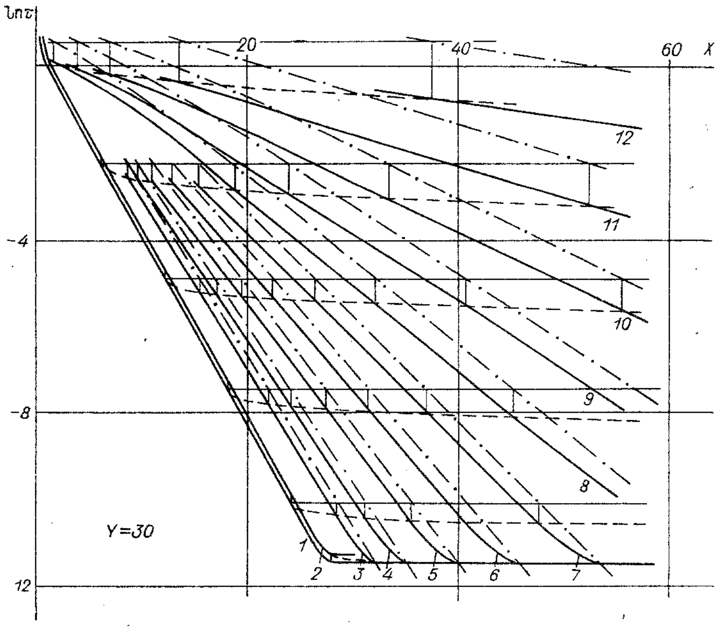

Fig. 4

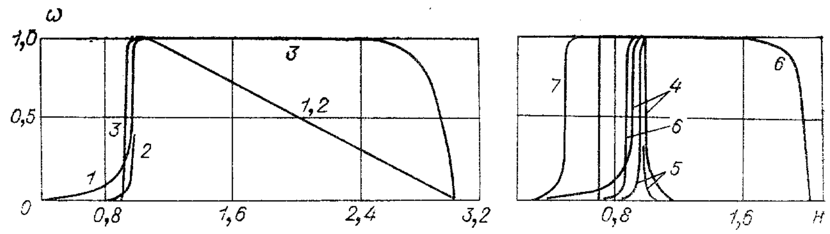

Fig. 5

of applying a kinetic approach, are confirmed qualitatively by a fractographic portrait of such brittle materials as certain plastics and glass [9] for which the spall surface is smooth, and viscous materials such as soft steel, e.g., [10] for which the spall can turn out to be uneven (because of the presence of a certain initial material vulnerability), or appear in the form of numerous coplanar cracks.

The kinetic approach permits setting up a relation between the magnitudes of the initial phase $I = XT/2$ and the spall enclosed in the piece of material being spalled ($0 \leqslant h \leqslant H$), $I_0 = \int_{\tau_c}^{\infty} X(h, \tau)\, d\tau$.

Three spall pulse shapes are possible (see Fig. 3), to which the values of the integrals correspond

$$I_0 = \frac{XT}{2}\frac{H^2}{2}, \quad 0 < \tau_p \leqslant T(1-H),$$

$$I_0 = \frac{XT}{2}\frac{H^2}{2}\left\{1 - 2\left[1 - \frac{1}{H}\left(1 - \frac{\tau_p}{T}\right)\right]^2\right\}, \quad T(1-H) \leqslant \tau_p \leqslant T\left(1 - \frac{H}{2}\right),$$

$$I_0 = \frac{XT}{2}\left(1 - \frac{\tau_p}{T}\right)^2, \quad T\left(1 - \frac{H}{2}\right) \leqslant \tau_p \leqslant T.$$

In conformity with computations performed by means of (2) for $H < 0.95$, the first spall pulse shape is realized for $6 \leqslant X \leqslant 24$, for which the ratio between the spall and initial pulses I_0/I as a function of H grows as a square law. The ratio I_0/I later passes through a maximum on the order of 0.46-0.48 (depending on the values of X and H), and drops sharply to zero for $H = 1$ (the relationship $I_0/I \leqslant 0.5$ is satisfied for a rectangular compression pulse).

On the other hand, it can be shown from the results obtained that a pulse needed for spalling at a given distance $S = CTH/2\nu$ from the free surface will diminish with the growth of H, hence if the results of a computation are interpreted from the viewpoint of spall production and ground ejection, then pulses yielding spalls with high values $H \lesssim 1$ are optimal for this purpose.

Let us make still another remark in conclusion. As the results of computations show (see Figs. 2 and 4), the longevity emerges at a constant level corresponding to a specific value of Y. At the same time, experiments on spalling fracture show that [8, 11, 12] a certain dependence of the longevity on the stress is observed even in the limit load domain $X \approx Y$. The disagreement between computation and experiment can be associated with not taking into account the influence of the compressive stress of the initial pulse on the material properties in the model, as well as with the fact that under real conditions the tensile stresses in the sections do not grow by jumps, as is assumed in this paper, but on a linear growth section [11]. The possibility of introducing "strengthening" in the dependence of the energy reduction on the external stresses $\varphi(\sigma)$, and also the form of the longevity curve for a linearly increasing load $X = V\tau$ (curve 3 in Fig. 2, $Y = 30$) show that the kinetic model under consideration is capable of easily eliminating the discrepancy mentioned.

The results obtained indicate that kinetic equations of the type (2) can even be applied to a problem about spalling fracture for a complex stress state of the material.

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